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LETTER TO THE EDITOR

Scaling features for the surface of ion-induced fractals

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Abstract. Scaling properties of the growth probability distribution were studied for ion-induced fractals in Ni-Zr alloy thin films when the irradiation dose reached a critical value. Two kinds of probability measure were used in our analysis, namely growth-site area obtained from DLA simulation of the growth process, and the Laplacian potential gradient. The generalised dimensions $D(q)$ with q ranging from $-\infty$ to $+\infty$, as well as the f-α spectrum were found to be slightly different from those obtained by other authors.

In recent years, fractal phenomena have been extensively studied by computer simulation and some experimental methods [1-5]. The diffusion-limited aggregation (DLA) model, which was first introduced by Witten and Sander [1], has proven valuable in describing a wide range of physical as well as chemical phenomena. According to recent significant progress on fractals, it is clear that a fractal cannot be completely characterised by its fractal dimension alone. For instance, some fractals, such as DLA aggregates and percolating clusters, have completely different structures, though their fractal dimensions are nearly the same. A new concept of multifractals was therefore introduced by Mandelbrot [6] and further developed by Halsey *et al* [7-10]. Accordingly, a fractal is considered as a hierarchical structure, and a fractal set is divided into a family of subsets, of which each subset has its own fractal dimension and singularity. Consequently, a fractal is characterised by a family of fractal dimensions. In this letter, we present the results of our recent study of the multifractalities of the fractals observed in a real two-dimensional physical system.

First of all, we briefly describe observations of fractal patterns induced in Ni-Zr thin solid films by ion irradiation, as the observed fractal patterns are the starting point of our present study. Ni-Zr multilayered films were prepared by alternately depositing pure Ni and Zr metal layers, of which each layer was about 8-10 nm, and the total thickness was about 400 nm, which matched the range of irradiating ions (200 keV Xe^+). The average composition of the multilayers was approximately $Ni_{55}Zr_{45}$, obtained by adjusting the relative thickness of the metals. The irradiation was conducted at liquid nitrogen temperature, and the ion current density was maintained below $1\ \mu m\ cm^{-2}$ to avoid overheating effects. The essential point of our experiments was to irradiate the films to various doses, covering a wide range as well as fine intervals, in order to approach a critical state of phase transformation. The results showed that a dose of $7\times10^{14}\ Xe^+\ cm^{-2}$ was sufficient to amorphise the films uniformly, and that a dose of $9\times10^{14}\ Xe^+\ cm^{-2}$ was the critical dose, inducing amorphous structural transformation. Figure 1 shows the observed fractal pattern by TEM bright-field examination in a sample after irradiating to the critical dose. The pattern consisted

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Figure 1. Fractal pattern observed in Ni–Zr thin films by TEM bright-field examination. The mass dimension is 1.4 ± 0.1.

of many branches and each branch consisted of many single crystals identified to be NiZr intermetallic compound, while the matrix still remained amorphous. The dimension of this pattern was 1.4 ± 0.1 determined by image processing computer with a resolution of 512×512 pixels [11, 12].

In order to understand the growth features of the fractal patterns observed in the Ni–Zr alloy system, we studied the multifractalities of these fractals. Since the growth process of the ion-induced fractal pattern cannot be observed directly by TEM, the methods employed by Ohta *et al* [13] cannot be employed in our case. We, therefore used computer simulation to study the growth processes of the fractal patterns. The DLA model was used in our simulation, based on the fact that the formation of the ion-induced fractal structure was controlled by the diffusion of the constituent atoms, i.e. Ni atoms and Zr atoms. The fractal structure was formed during the phase transformation from the amorphous mixture to the NiZr intermetallic compound. Since the average composition of the sample was approximately $Ni_{55}Zr_{45}$, it was necessary to have Ni and Zr atoms diffuse in order to form the equatomic NiZr phase.

One of the fractal patterns was first stored in the memory of a computer through a television image processor, and then displayed on the screen. A particle was set at a distance from the fractal pattern, and allowed to move randomly. When the particle reached the perimeter of the fractal pattern, a growth point was set at this place. If the particle strayed too far from the fractal, we stopped it and set another particle in motion. This process was repeated hundreds of times. Eventually, a pattern was obtained after simulating 1000 particles and is shown in figure 2.

In order to compare our results with those of other authors, and also to check the above method, the Laplacian potential gradient measure was used. The growth probability of each growth point was proportional to the Laplacian potential gradient at the point, i.e. $P(i,j)\propto|\nabla_n\phi|$. We solved the Laplace equation numerically on the two-dimensional square lattice by the relaxation method under the following conditions: the fractal structure itself was equipotential (the potential was set to 0); the outer

Figure 2. The growth pattern obtained from the computer simulation based on figure 1. The pattern consists of 1000 growth particles.

boundary was also equipotential (the potential was set to 1); the diameter of the outer boundary was about three times larger than the fractal diameter.

Now we cover the perimeter of the fractal structure with boxes. In the first case, let $P_i(\varepsilon)$ refer to the number of growth points within the ith box of size ε. In the second case let $P_i(\varepsilon)$ be the sum of growth probabilities within the ith box. Apparently, $P_i(\varepsilon)$ is just the growth probability accumulated in the ith box. Note that both of the probabilities were normalised. For our fractal structures, the size of boxes ε ranged from 10 pixels to 40 pixels. The generalised dimensions $D(q)$ can be calculated according to the following definition:

$$D(q) = \lim_{\varepsilon \to 0} (q-1)^{-1} \log\left(\sum_i (P_i(\varepsilon))^q\right) (\log(\varepsilon))^{-1} \tag{1}$$

$$D_{(1)} = \lim_{\varepsilon \to 0} \left(\sum_i P_i(\varepsilon) \log(P_i(\varepsilon))\right) (\log(\varepsilon))^{-1}. \tag{2}$$

The f-α spectra were also obtained from the following equations:

$$\alpha(q) = \frac{\mathrm{d}}{\mathrm{d}q}(q-1)D(q) \tag{3}$$

$$f(\alpha(q)) = q\alpha(q) - (q-1)D(q). \tag{4}$$

The results are shown in figures 3 and 4.

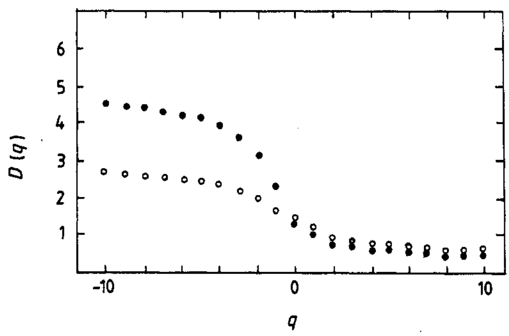

Figure 3. The generalised dimensions $D(q)$ plotted against q. The full circles plot the growth-site area measure, and the open circles plot the Laplacian potential gradient measure.

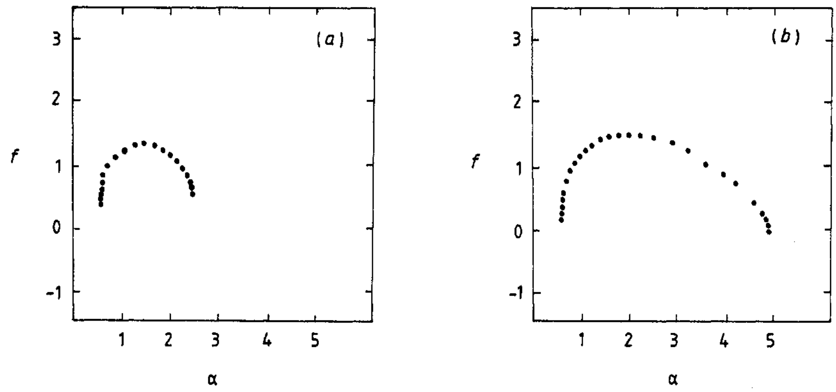

Figure 4. The f–α spectra obtained from (a) the growth-site area measure, and (b) the Laplacian potential gradient measure.

From figures 3 and 4, one can see that there are some differences between the results obtained using the two different methods. Particularly, in the negative q direction, the difference is large. The generalised dimensions $D(q)$ obtained using DLA simulation are much smaller than those obtained using the Laplacian potential gradient measure when q is negative. ($D(-\infty)$ is 2.7 for the first measure, and 4.6 for the second measure.) This is reasonable, since some strongly screened growth points, which make a large contribution to $D(q)$ when q is negative, have very small growth probabilities and are impossible to grow. (Figure 2 supports this explanation.) Hence, the results from DLA simulation seem more realistic. However, when $q > 0$, the difference is very small. ($D(+\infty)$ is 0.8 for the first measure, and 0.7 for the second measure.) $D(0)$ equals 1.40, which is just the determined mass dimension of the fractal structure. The information dimension $D(1)$ is 1.1, which agrees with the theoretical result.

In conclusion, we proposed a new method for studying the multifractality of a fractal structure, whose growth process is not easy to observe directly. The Laplacian potential gradient measure was also used to perform the analysis. The results from different measures are qualitatively consistent with each other; however, there exist some quantitative differences. Both of the results confirmed that the fractal structure observed in Ni–Zr thin films kept multifractal structures. A similar study was also conducted for ion-induced fractals in Ni–Mo [14] thin films, and similar results will be published in a forthcoming paper.

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